

Non-extensive statistics, fluctuations and correlations in high-energy nuclear collisions

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Abstract. Starting from the experimental evidence that high-energy nucleus–nucleus collisions cannot be described in terms of superpositions of elementary nucleon–nucleon interactions, we analyze the possibility that memory effects and long-range forces imply a non-extensive statistical regime during high-energy heavy-ion collisions. The relevance of these statistical effects and their compatibility with the available experimental data are discussed. In particular, we show that theoretical estimates obtained in the framework of the generalized non-extensive thermostatistics, can reproduce the shape of the pion transverse mass spectrum and explain the different physical origin of the transverse momentum correlation function of the pions emitted during the central Pb + Pb and during the $p + p$ collisions at 158 GeV.

1 Introduction

In the last years, many efforts have been focussed on the study of high-energy nuclear collisions, resulting in a better understanding of the strongly interacting matter at high energy density. Large-acceptance detectors provide a detailed analysis of the multiplicity of the particles produced; this in turn can be related to possible signatures of the formation of a new phase of matter, the so-called quark–gluon plasma (QGP), in the early stages of high-energy heavy-ion collisions [1].

One of the most interesting experimental results is that high-energy nucleus–nucleus ($A + A$) collisions cannot be described in terms of superpositions of elementary nucleon–nucleon ($N + N$) interactions (proton–proton or proton–antiproton). The different conditions of energy density and temperature at the early stage of the Pb + Pb and of the $p + p$ collisions (to mention just two extreme examples) generate collective effects that modify the features of freeze-out observables. Of course, the most appealing explanation would be to interpret the presence of these evident experimental differences as an indirect consequence of the formation of QGP in the early stages of heavy-ion collisions.

It is an experimentally established fact [2] that the slope parameter of the transverse momentum distribution significantly increases with the particle mass when going from $p + p$ to central Pb + Pb collisions. On the other hand, the preliminary results of the NA49 Collaboration [3, 4] indicate that the transverse momentum fluctuations in central Pb + Pb collisions at 158 GeV have a quite different physical origin with respect to the corresponding fluctua-

tions in $p + p$ collisions. Let us also recall that the observed J/Ψ suppression increases continuously and monotonically from the lightest ($p + p$) to the heaviest (S + U) interacting nuclei, but it exhibits a clear departure from this “normal” behavior for central Pb + Pb collisions [5].

These experimental features have been explained in terms of collective effects in the hadronic medium (such as the presence of a transverse hydrodynamical expansion), and the strong influence of secondary rescatterings in heavy-ion collisions has been considered [2, 6–8]. It is a very important issue to understand whether the system is able to reach full thermalization; up to now this question has been the subject of many experimental and theoretical studies. However, several experimental observables, such as the transverse mass spectrum, the multiplicity of particles and the three-dimensional phase-space density of pions, are well reproduced in the framework of local thermal equilibrium in a hydrodynamical expanding environment [8–12].

In this paper we start from the hypothesis that memory effects and long-range forces can occur during high-energy heavy-ion collisions: in this case the strong influence of collective effects on the experimental observables can be understood in a very natural way in the framework of the generalized non-extensive thermostatistics. In this context we show that the transverse momentum correlations of the pions emitted from central Pb + Pb collisions can be very well reproduced by means of very small deviations from the standard equilibrium extensive statistics.

In Sect. 2 we summarize the main assumptions contained in the derivation of the dynamical kinetical equa-

tions and their relevance to the determination of the equilibrium phase-space distribution. In Sect. 3 we examine whether these conditions are met in the high-energy heavy-ion collisions and which experimental observables can be sensitive to the presence of non-extensive statistical effects. In Sect. 4 we introduce the non-extensive Tsallis thermostatics which is considered the natural generalization of the standard classical and quantum statistics when memory effects and/or long-range forces are not negligible. In Sect. 5 we show how the generalized distribution function modifies the shape of the transverse mass spectrum and in Sect. 6 we investigate the influence of non-extensive particle fluctuations in the determination of the measure of the transverse momentum correlations. Finally, a discussion of the results and some conclusions are reported in Sect. 7.

2 Dynamical kinetical approach to thermal equilibrium and extensive statistics

Assuming local thermal equilibrium, the freeze-out observables are usually calculated within the extensive thermostatics. In particular, the equilibrium transverse momentum distribution is assumed to be, in the classical case, the standard Maxwell–Boltzmann (Jüttner) distribution. If the emitted particles are light enough, such as pions, to have noticeable quantum degeneracy, one must use the Bose–Einstein (or Fermi–Dirac, for fermions) distribution.

When the system approaches equilibrium, the phase-space distribution should be derived as a stationary state of the dynamical kinetical evolution equation. If, for the sake of simplicity, we limit our discussion to the classical case, the Maxwellian distribution is obtained as a steady state solution of the Boltzmann equation. Let us now briefly review the principal assumptions in the derivation of the Boltzmann equation and examine how these approximations can affect the determination of the equilibrium distribution [13]. One important assumption concerns the collision time, which must be much smaller than the mean time between collisions. This condition can be expressed by $nr_0^3 \ll 1$, where n is the density and r_0 is the effective range of the interactions. This condition has two important physical consequences.

- (1) There is no overlapping between subsequent collisions involving a given particle and the interactions can be described as a succession of simple binary collisions.
- (2) It is always possible to define a time interval in which the single particle distribution does not change appreciably and its rate of change at time t depends only on its instantaneous value and not on its previous history. Hence this property reflects the Markovian character of the Boltzmann equation: no memory is taken into account.

The second assumption is analogous to the first one, but for the space dependence of the distribution function: its rate of change at a spatial point depends only on the neigh-

borhood of that point. In other words the range of the interactions is short with respect to the characteristic spatial dimension of the system.

The last important assumption is the so-called Boltzmann Stosszahlansatz: the momenta of two particles at the same spatial point are not correlated and the corresponding two-body correlation function can be factorized as a product of two single particle distributions. This assumption is very important because it allows us to write down the collisional integral (which is equal to the total time rate of change of the distribution function) in terms of the single particle distribution only. The above assumptions, namely the absence of non-Markovian memory effects, the absence of long-range interactions and negligible local correlations, together with the Boltzmann’s H theorem (based on the *extensive* definition of the entropy) lead us to the well-known stationary Maxwellian distribution.

The basic assumption of standard statistical mechanics is that the system under consideration can be subdivided into a set of non-overlapping subsystems. As a consequence the Boltzmann–Gibbs entropy is extensive in the sense that the total entropy of two independent subsystems is the sum of their entropies. If memory effects and long-range forces are present, this property is no longer valid and the entropy, which is a measure of the information about the particle distribution in the states available to the system, is not an extensive quantity.

In the next section, we will see whether the above assumptions are implemented during high-energy nuclear collisions and if there appear experimental signals that can be interpreted as a consequence of the presence of a non-extensive regime.

3 Do heavy-ion collisions satisfy extensive statistics?

It is a rather common opinion that, because of the extreme conditions of density and temperature in ultrarelativistic heavy-ion collisions, memory effects and long-range color interactions give rise to the presence of non-Markovian processes in the kinetic equation affecting the thermalization process toward equilibrium as well as the standard equilibrium distribution [14–17].

A rigorous determination of the conditions that produce a non-extensive behavior, due to memory effects and/or long-range interactions, should be based on microscopic calculations relative to the parton plasma originated during the high-energy collisions. At this stage we limit ourselves to consider the problem from a qualitative point of view on the basis of the existing theoretical calculations and experimental evidences.

Under the hypothesis that QGP is generated in the high-energy collisions, the quantities characterizing the plasma, such as lifetime and damping rates of quasiparticles, are usually calculated within finite temperature perturbative QCD. This approach is warranted by the fact that at high temperature (beyond a few hundred MeV) the strong QCD coupling $\alpha_s = g^2/4\pi$ becomes very small

and “weak coupling” regime takes place. In this regime ($g \ll 1$) the longitudinal gluon propagator is characterized by a Debye mass $m_D \approx gT$, and the corresponding screening length ($\lambda_D = m_D^{-1} \approx 1/gT$) is much larger than the mean interparticle distance ($\langle r \rangle \approx n^{-1/3} \approx 1/T$) (for a review see, e.g., [18]). Therefore a large number of partons is contained in the Debye sphere and the ordinary mean field approximation of QGP (Debye–Hückel theory) holds. Nevertheless, in the proximity of the phase transition, the QGP is no longer a system of weakly interacting particles and non-perturbative QCD calculations become important. Recently, it has been shown [19] that for a temperature near the critical one ($T \geq 1.1T_c$) non-perturbative calculations imply an effective quark mass very close to T , sensibly different from the value, proportional to gT , obtained in perturbative regime. Near the phase transition the characteristic quantity $nr_0^3 \approx n\lambda_D^3$ should then be very close to one and only a small number of partons is present in the Debye sphere: the ordinary mean field approximation of the plasma is no longer correct and memory effects are not negligible. In addition, we observe that in high density quark matter the color magnetic field remains unscreened (in leading order) and long-range color magnetic interaction should be present at all temperatures.

From the above considerations it appears reasonable that, if the deconfining phase transition takes place, non-Markovian effects and long-range interactions can influence the dynamical evolution of the generated fireball toward the freeze-out stage. Moreover, they will affect the equilibrium phase-space distribution function if thermal equilibrium is attained. A signature of these effects should show up in physical observables. In this context, we notice that the authors of [7] raise a controversy on the Markovian description of the multiple scattering processes, one of the assumptions of the UrQMD model. This example concerns $e^+e^- \rightarrow \mu^+\mu^-$ scattering, where a determination of the angular distribution based on the Markovian approximation of the transport theory leads to wrong results [20, 21, 17].

The aim of the present work is to suggest a possible interpretation of the above-mentioned difference between the correlations/fluctuations of the pion transverse momentum measured in $p + p$ and Pb + Pb collisions [11] as a signature of the non-extensivity of the system. In fact, it has been shown [22] that, whether the equilibrium condition is realized or not, the measure of the correlations/fluctuations strongly depends on the factorization of the multiparticle distributions and on the event by event multiplicity of the particles under consideration. Such a property implies that the (multi)particle distribution functions are not inclusive (the distribution functions of one, two, or more bodies depend on the presence of the other particles of the system). In turn, this is a manifestation of a non-extensive behavior of the system.

In addition we notice that, from a recent analysis of the average pion phase-space density at freeze-out in S-nucleus and Pb–Pb collisions at SPS, the experimental data indicate a slower decrease with increasing p_\perp than

the Bose–Einstein curve [23]. The analysis of the same quantity in π – p collisions seems, instead, to be consistent with the standard expectations. We recall that in [24] the authors show that by assuming the standard Jüttner momentum distribution of emitted particles in high-energy collisions one is led to unrealistic consequences and the physical freeze-out is actually not realized.

From the foregoing considerations we conclude that both theoretical calculations and experimental observables agree with the existence of non-extensive features in high-energy heavy-ion collisions.

4 Generalized non-extensive Tsallis statistics

Several new developments in statistical mechanics have shown that in the presence of long-range forces and/or in irreversible processes related to microscopic long-time memory effects, the extensive thermodynamics, based on the conventional Boltzmann–Gibbs thermostatics, is no longer correct and, consequently, the equilibrium particle distribution functions can show different shapes from the conventional well-known distributions. Standard sums, or integrals, which appear in the calculation of thermostatical quantities, like the partition function, the entropy, the internal energy, etc. can diverge. These difficulties have been well known in several physical domains [25] since a long time.

A quite interesting generalization of the conventional Boltzmann–Gibbs statistics has been recently proposed by Tsallis [26] and proves to be able to overcome the shortcomings of the conventional statistical mechanics in many physical problems, where the presence of long-range interactions, long-range microscopic memory, or fractal space-time constraints hinders the usual statistical assumptions. Among other applications, we quote astrophysical self-gravitating systems [27], the solar neutrino problem [28], cosmology [29], many-body, dynamical linear response theory and variational methods [30] and phase shift analyses for the pion–nucleus scattering [31].

The Tsallis generalized thermostatics is based upon the following generalization of the entropy [26]

$$S_q = \frac{1}{q-1} \sum_{i=1}^W p_i (1 - p_i^{q-1}), \quad (1)$$

where p_k is the probability of a given microstate among W different ones and q is a fixed real parameter.

The new entropy has the usual properties of positivity, equiprobability, concavity and irreversibility, preserves the whole mathematical structure of thermodynamics (Legendre transformations) and reduces to the conventional Boltzmann–Gibbs entropy $S = -\sum_i p_i \log p_i$ in the limit $q \rightarrow 1$.

The deformation parameter q measures the degree of non-extensivity of the theory. In fact, if we have two independent systems A and B , such that the probability of $A + B$ is factorized into $p_{A+B}(u_A, u_B) = p_A(u_A)p_B(u_B)$; the global entropy is not simply the sum of their entropies

but it is easy to verify that

$$S_q(A+B) = S_q(A) + S_q(B) + (1-q)S_q(A)S_q(B). \quad (2)$$

For a better appreciation of the meaning of non-extensivity in a physical system, we discuss here another important property. Let us suppose that the set of W microstates is arbitrarily separated into two subsets having W_L and W_M microstates ($W_L + W_M = W$), respectively, and define as $p_L \equiv \sum_{i=1}^{W_L} p_i$ and $p_M \equiv \sum_{i=W_L+1}^W p_i$ the corresponding probabilities: hence $p_L + p_M = 1$. It is then easy to show that

$$S_q(\{p_i\}) = S_q(p_L, p_M) + p_L^q S_q(\{p_i/p_L\}) + p_M^q S_q(\{p_i/p_M\}), \quad (3)$$

where the sets $\{p_i/p_L\}$ and $\{p_i/p_M\}$ are the conditional probabilities. This is a generalization of the famous Shannon's property but for the appearance of p_L^q and p_M^q instead of p_L and p_M in the second and third terms of the right hand side of (3). Since the probabilities $\{p_i\}$ are normalized, $p_i^q > p_i$ for $q < 1$ and $p_i^q < p_i$ for $q > 1$: as a consequence values of $q < 1$ ($q > 1$) will favor rare (frequent) events, respectively.

The single particle distribution function is obtained through the usual procedure of maximizing the Tsallis entropy under the constraints of keeping constant the average internal energy and the average number of particles. For a dilute gas of particles and/or for $q \approx 1$ values, the average occupational number can be written in a simple analytical form [32]

$$\langle n_i \rangle_q = \frac{1}{[1 + (q-1)\beta(E_i - \mu)]^{1/(q-1)} \pm 1}, \quad (4)$$

where the + sign is for fermions, the - for bosons and $\beta = 1/T$. In the limit $q \rightarrow 1$ (extensive statistics), one recovers the conventional Fermi-Dirac and Bose-Einstein distribution.

Let us recall that the Tsallis generalized statistics does not entail a violation of the Pauli exclusion principle and does not modify the inclusive behavior of the bosons, but it modifies, with the generalized non-extensive entropy (1), the extensive nature of the standard statistics. At the equilibrium, the non-extensive statistics implies a finite temperature particle distribution different from the standard Fermi-Dirac and Bose-Einstein distributions; in the classical limit, one has the following generalized Maxwell-Boltzmann distribution [26]:

$$\langle n_i \rangle_q = [1 + (q-1)\beta(E_i - \mu)]^{1/(1-q)}. \quad (5)$$

When the entropic q parameter is smaller than 1, the distributions (4) and (5) have a natural high-energy cut-off: $E_i \leq 1/[\beta(1-q)] + \mu$, which implies that the energy tail is depleted; when q is greater than 1, the cut-off is absent and the energy tail of the particle distribution (for fermions and bosons) is enhanced. Hence the non-extensive statistics entails a sensible difference of the particle distribution shape in the high-energy region with respect to the standard statistics. This property plays an important rôle in the interpretation of the physical observables, as will be shown in the following.

5 Transverse mass spectrum and q blue shift

The applicability of the equilibrium statistical mechanics relies on the recognition that the experimental transverse mass spectrum and the relative abundance of charged particles (pions, kaons, protons, etc.) can be described within the equilibrium formalism. The single particle spectrum can be expressed as an integral over a freeze-out hypersurface Σ_f [33]

$$E \frac{d^3 N}{d^3 p} = \frac{dN}{dy m_\perp dm_\perp d\phi} = \frac{g}{(2\pi)^3} \int_{\Sigma_f} p^\mu d\sigma_\mu(x) f(x, p), \quad (6)$$

where g is the degeneracy factor and $f(x, p)$ is the phase-space distribution.

Indeed, by assuming a purely thermal source with a Boltzmann distribution, the transverse mass spectrum can be expressed as follows:

$$\frac{dN}{m_\perp dm_\perp} = A m_\perp K_1(z), \quad (7)$$

where $z = m_\perp/T$ and K_1 is the first order modified Bessel function. In the asymptotic limit, $m_\perp \gg T$ ($z \gg 1$), the above expression gives rise to the exponential shape

$$\frac{dN}{m_\perp dm_\perp} = B \sqrt{m_\perp} e^{-z}, \quad (8)$$

which is usually employed to fit the experimental transverse mass spectra and provides an indication of a thermal energy distribution.

Although, from a general perspective, the experimental distribution of particle momenta in the transverse direction is described by an exponentially decreasing behavior, at higher energies the experimental data deviate from the usual Boltzmann slope. This discrepancy can be cured by introducing the dynamical effect of a collective transverse flow which predicts the so-called blue shift factor, i.e., an increase of the slope parameter T at large m_\perp .

The experimental slope parameter measures the particle energy, which contains both thermal (random) and collective (mainly due to rescattering) contributions. The thermal motion determines the freeze-out temperature T_{fo} , namely the temperature when the particles cease to interact with each other. In the presence of a collective transverse flow [2] one can extract T_{fo} from the empirical relation

$$T = T_{fo} + m \langle v_\perp \rangle^2, \quad (9)$$

where $\langle v_\perp \rangle$ is a fit parameter which can be identified with the average collective flow velocity; the latter is consistent with zero in $p+p$ reactions, but can be large in collisions between heavy nuclei (where rescattering is expected to be important). In (9) m is the mass of the detected particle (e.g. pions, kaons, or protons).

Let us consider a different point of view and argue that the deviation from the Boltzmann slope at high p_\perp can be ascribed to the presence of non-extensive statistical effects in the steady state distribution of the particle gas; the latter do not exclude the physical effect of a collective

flow but rather incorporates a description of it from a slightly different statistical mechanics analysis.

If long tail time memory and long-range interactions are present, as already discussed in Sect. 3, the Maxwell–Boltzmann distribution must be replaced by the generalized distribution (5). We consider here only small deviations from standard statistics ($q - 1 \approx 0$); then at first order in $(q - 1)$ the transverse mass spectrum can be written as

$$\frac{dN}{m_{\perp} dm_{\perp}} = C m_{\perp} \left\{ K_1(z) + \frac{(q-1)}{8} z^2 [3K_1(z) + K_3(z)] \right\}, \quad (10)$$

where K_3 is the modified Bessel function of the third order. In the asymptotic limit, $z \gg 1$ (but always for q values close to 1, such that $(q - 1)z \ll 1$), we obtain the following generalization of (8):

$$\frac{dN}{m_{\perp} dm_{\perp}} = D \sqrt{m_{\perp}} \exp\left(-z + \frac{q-1}{2} z^2\right). \quad (11)$$

From the above equation it is easy to see that at first order in $(q - 1)$ the generalized slope parameter becomes the quantity T_q , defined as

$$T_q = T + (q - 1)m_{\perp}. \quad (12)$$

Hence non-extensive statistics predicts, *in a purely thermal source*, a generalized q blue shift factor at high m_{\perp} ; moreover, this shift factor is not constant but increases (if $q > 1$) with $m_{\perp} = (m^2 + p_{\perp}^2)^{1/2}$, where m is the mass of the detected particle. We have in this case an effect very similar to the one described by the phenomenological relation (9). We remark that the proposed distribution of (11), based on the non-extensive thermostatics, has the same high p_{\perp} dependence of the parameterization reported in [39] to fit the transverse distribution for central Pb + Pb collisions.

The value of the entropic parameter q is a measure of the non-extensivity of the system and, as a consequence, it should be fixed by the reaction under consideration. On the basis of the above considerations, at a fixed reaction energy, one should find that larger values of q are required for heavier colliding nuclei, because at higher energy density we have a larger probability for the phase transition to be achieved.

A quantitative description of the particle production in non-extensive statistics goes beyond of the scope of this paper; it is fair to assume that (11), for $q > 1$ value, can be in good agreement with experimental results in heavy-ion collisions. In this framework, indeed, one can understand the slower decrease (for heavier colliding nuclei) of the pion phase-space distribution with increasing p_{\perp} .

It is worth noticing that for small deviations from the standard statistics ($|(q - 1)| < 0.05$) the experimental midrapidity transverse mass distributions are well reproduced with the same slope parameter of [2], within the statistical errors.

In the next section we shall calculate the measure of the transverse momentum fluctuations in the framework of the equilibrium non-extensive statistics. We notice that, since small non-extensive statistical effects are consistent with the experimental transverse mass spectrum, we can use, in what follows, the same temperature extracted from the standard analysis of the experimental results.

6 Transverse momentum fluctuations in non-extensive statistics

In [34], Gaździcki and Mrówczyński have introduced a quantity Φ , which measures the event-by-event fluctuations and does not explicitly depend on the particle multiplicity, but appears to be sensitive to the correlations. It is defined as

$$\Phi_x = \sqrt{\frac{\langle Z_x^2 \rangle}{\langle N \rangle}} - \sqrt{\frac{z_x^2}{z_x}}, \quad (13)$$

where x is a variable such as energy or transverse momentum, $z_x = x - \bar{x}$ is a single particle variable and $Z_x = \sum_{i=1}^N (x_i - \bar{x})$ is the corresponding sum variable, N being the number of the particles in the event.

A non-vanishing value of the measure Φ implies that there is an effective correlation among particles (dynamical or statistical) which alters the momentum distribution. It is usually assumed that if the experimental value of Φ turns out to be constant in going from $N + N$ to $A + A$ collisions, then nucleus–nucleus collisions can be described as an incoherent superposition of elementary $N + N$ scattering processes and the correlations among particles are unchanged [22,34]. We wish here to point out that this might not be the case: indeed it will be shown that similar values of Φ can be attributed to different statistical correlations, which in turn reflect distinct dynamical situations.

Recently, the correlation measure $\Phi_{p_{\perp}}$ of the pion transverse momentum has been measured by the NA49 Collaboration. The experimental results correspond to a value $\Phi_{p_{\perp}}^{\text{exp}} = 0.6 \pm 1 \text{ MeV}$ for Pb + Pb at 158 GeV and to $\Phi_{p_{\perp}}^{\text{exp}} = 5 \pm 1 \text{ MeV}$ in the preliminary measurements of the $p + p$ collisions at the same energy [3,4]. However, in their most recent analysis of the Pb + Pb data, the NA49 Collaboration has estimated a contribution of $\Delta\Phi_{p_{\perp}} = 5 \pm 1.5 \text{ MeV}$ from the statistical two particle correlation function alone and an “anti-correlation” contribution of $\Delta\Phi_{p_{\perp}} = -4 \pm 0.5 \text{ MeV}$, stemming from the limitation in the two-track resolution of the NA49 apparatus [4]. Hence it appears that the actual physical values both for $p + p$ and for Pb + Pb collisions are nearly equal. In any case, as was observed in the same paper, the physical origin of transverse momentum fluctuation remarkably changes from Pb + Pb to $p + p$ collisions and, although the pion transverse correlation measure has been studied within many different theoretical approaches, the obtained results are somewhat controversial and not well understood [6,35–38].

We will show in the following that the physical differences in the origin of the pion transverse momentum fluctuations in Pb + Pb collisions with respect to the $p + p$ ones

can be understood in the framework of the non-extensive statistics. For this purpose, keeping in mind that the whole mathematical structure of the thermodynamical relations is preserved in the non-extensive statistics, it is easy to show that the two terms in the right hand side of (13) can be expressed in the following simple form:

$$\overline{z_{p_\perp}^2} = \frac{1}{\rho} \int \frac{d^3p}{(2\pi)^3} (p_\perp - \bar{p}_\perp)^2 \langle n \rangle_q, \quad (14)$$

and

$$\frac{\langle Z_{p_\perp}^2 \rangle}{\langle N \rangle} = \frac{1}{\rho} \int \frac{d^3p}{(2\pi)^3} (p_\perp - \bar{p}_\perp)^2 \langle \Delta n^2 \rangle_q, \quad (15)$$

where the quantity \bar{p}_\perp is the average transverse momentum,

$$\bar{p}_\perp = \frac{1}{\rho} \int \frac{d^3p}{(2\pi)^3} p_\perp \langle n \rangle_q \quad \text{with} \quad \rho = \int \frac{d^3p}{(2\pi)^3} \langle n \rangle_q. \quad (16)$$

In the above equations we have indicated with $\langle n \rangle_q$ the mean occupation number (4) extended to the continuum and with $\langle \Delta n^2 \rangle_q = \langle n^2 \rangle_q - \langle n \rangle_q^2$ the generalized particle fluctuations, given by

$$\langle \Delta n^2 \rangle_q \equiv \frac{1}{\beta} \frac{\partial \langle n \rangle_q}{\partial \mu} = \frac{\langle n \rangle_q}{1 + (q-1)\beta(E-\mu)} (1 \mp \langle n \rangle_q), \quad (17)$$

where E is the relativistic energy $E = (m^2 + p^2)^{1/2}$. For $q = 1$, one recovers the well-known text-book expression for the fluctuations of fermions ($-$) and bosons ($+$). Equation (15) shows that the correlation measure Φ arises from the particle fluctuations; this result does not come as a surprise since it is well known that the particle fluctuations are related to the two-body density or to the isothermal compressibility by means of the so-called Ornstein–Zernike relation [13].

In the standard extensive statistics ($q = 1$), one obtains that the correlation measure Φ is always negative for fermions and positive for bosons, while it vanishes in the classical standard statistics [35]. Indeed $\langle Z^2 \rangle$ is defined in terms of the fluctuations Δn^2 , which for the ideal Fermi–Dirac (Bose–Einstein) gas are suppressed (enhanced) with respect to the classical case. In the non-extensive Tsallis statistics, the fluctuations of an ideal gas of fermions (bosons), expressed by (17), are still suppressed (enhanced) by the factor $1 \mp \langle n \rangle_q$; this effect, however, is modulated by the factor $[1 + (q-1)\beta(E-\mu)]^{-1}$ in the right hand side of (17). Therefore, the fluctuations turn out to be increased for $q < 1$ (we remind the reader that due to the energy cut-off condition the above factor is always positive) and are decreased for $q > 1$. The measure Φ does not have, both for bosons and fermions, a constant sign, and it is positive or negative depending on the value of the entropic parameter q . In the classical non-extensive case we also have $\langle \Delta n^2 \rangle_q \neq \langle n \rangle_q$. The quantity $\langle \Delta n^2 \rangle_q$ is greater than $\langle n \rangle_q$ for $q < 1$, and smaller for $q > 1$ and the correlation measure Φ can be different from zero for $q \neq 1$. This is essentially due to the intrinsic nature of the system

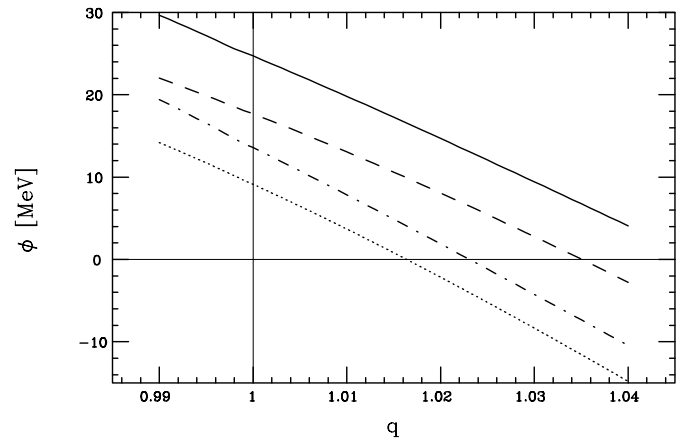


Fig. 1. The correlation measure Φ_{p_\perp} [MeV] as a function of the entropic parameter q , calculated in four different conditions of temperature T and chemical potential μ ($T = 170$ MeV and $\mu = 60$ MeV solid line, $T = 140$ MeV and $\mu = 60$ MeV dashed line, $T = 170$ MeV and $\mu = 0$ MeV dot-dashed line, $T = 140$ MeV and $\mu = 0$ MeV dotted line). The value $\Phi_{p_\perp}(q = 1)$ corresponds to the correlation measure calculated within the standard extensive statistics

in the presence of memory effects and/or long-range forces and to the dependence of the (multi)particle distribution functions upon the surrounding medium.

We use (14) and (15) to evaluate the correlation measure Φ_{p_\perp} for the pion gas ($m_\pi = 140$ MeV), which we can compare with the experimental results measured in the central Pb + Pb collision by the NA49 collaboration [3, 4]. Let us notice that the values of Φ_{p_\perp} , which depends on T , μ and the entropic parameter q , are very sensitive to small variations of q . This is mainly due to the modified expression (17) of the fluctuations in the non-extensive statistics.

In Fig. 1 we show Φ_{p_\perp} as a function of the parameter q , at different (fixed) values of the temperature and the chemical potential. The experimental data are reported up to a maximum value of the transverse momentum of $p_\perp = 1.5$ GeV: hence we have limited the integration up to the upper limit of 1.5 GeV. Concerning the temperature T , we use the slope parameter derived from the fit of the transverse momentum spectra [2, 39]. With $T = 170$ MeV and $\mu = 60$ MeV (according to the suggestion of [40]), we obtain $\Phi_{p_\perp} = 5$ MeV with $q = 1.038$, while the usual extensive statistics, with $q = 1$, gives $\Phi_{p_\perp} = 24.7$ MeV (we recall that a non-vanishing value of μ implies that there is no chemical equilibrium).

By choosing $\mu = 0$ MeV one can obtain the same value $\Phi_{p_\perp} = 5$ MeV with a smaller q ($q = 1.015$). In this case, with $q = 1$, $\Phi_{p_\perp} = 13.6$ MeV.

Thus, a very little deviation from the standard Bose–Einstein distribution is sufficient to obtain theoretical estimates in agreement with the experimental results. We notice also that the value employed, $q \approx 1$, justifies, a posteriori, the approximation of the analytical form of the distribution function (4) and the validity of (10) for

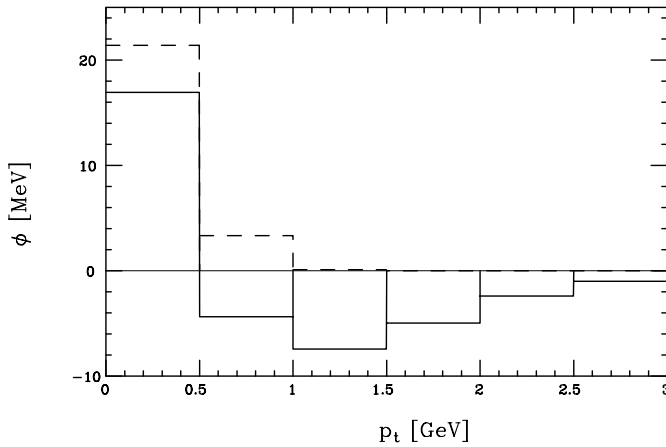


Fig. 2. The partial contributions to the correlation measure $\Phi_{p_{\perp}}$ [MeV] in different p_{\perp} intervals, at $T = 170$ MeV and $\mu = 60$ MeV. The dashed line refers to standard statistical calculations with $q = 1$, the solid line corresponds to $q = 1.038$ (note that by summing the contributions up to $p_{\perp} = 1.5$ GeV we obtain the value $\Phi_{p_{\perp}} = 5$ MeV, indicated by the NA49 experiment [4]). Negative contributions, at high p_{\perp} steps, are predicted only within the non-extensive statistics

the transverse mass spectrum. The correlations are much more affected by deviations from the extensive statistics than the single particle observables, such as the transverse mass distribution. We can obtain a very good agreement with the experimental results of $\Phi_{p_{\perp}}$, still being consistent with the other single particle measurements.

In this context, it is worth noticing that, even for small deviations from the standard statistics, an appreciable difference in the distribution function and in the fluctuations (with respect to the standard ones) appears *especially* in the region of high transverse momenta. This behavior is consistent with the interpretation of the non-extensive statistics effects, since the detected high-energy pions should be essentially the primary ones, hence more sensitive to the early stage of the collisions. In order to show this fact, we have evaluated the partial contributions to the quantity $\Phi_{p_{\perp}}$, by using (14) and (15), and by extending the integration over p_{\perp} to partial intervals $\Delta p_{\perp} = 0.5$ GeV. The results are shown in Fig. 2, at $T = 170$ MeV and $\mu = 60$ MeV. In the standard statistics (dashed line), $\Phi_{p_{\perp}}$ is always positive and vanishes in the p_{\perp} intervals above ≈ 1 GeV. In the non-extensive statistics (solid line), instead, the fluctuation measure $\Phi_{p_{\perp}}$ becomes negative for p_{\perp} larger than 0.5 GeV and becomes vanishingly small only in p_{\perp} intervals above ~ 3 GeV.

In view of these considerations, we suggest an analysis of the Pb + Pb measurements which allows to disentangle the contributions to $\Phi_{p_{\perp}}$ in different p_{\perp} intervals. A negative value of $\Phi_{p_{\perp}}$ at high transverse momentum intervals could be clear evidence of the presence of a non-extensive regime.

7 Conclusions

The non-extensive statistics appears suitable to evaluate physical observables recently measured in heavy-ion collision experiments. The spectrum of the transverse momentum can be reproduced by means of the non-extensive distribution, which naturally takes into account typical collective effects in heavy colliding nuclei such as the increasing of the slope parameter with high p_{\perp} and high particle masses. The calculated correlation measure $\Phi_{p_{\perp}}$ agrees with the experimental value by considering a small deviation from the standard statistics. Within the non-extensive approach we have also found negative partial contributions to $\Phi_{p_{\perp}}$ at high p_{\perp} (larger than 0.5 GeV). In this regime the $\Phi_{p_{\perp}}$ value is not affected by resonance decays: hence an experimental confirmation of our prediction would be an unambiguous signal of the validity of non-extensive statistics in relativistic heavy-ion collisions.

The quantity q is not just a free parameter: it depends on the physical conditions generated in the reaction and, in principle, should be related to microscopic quantities (such as the mean interparticle interaction length, the screening length and the collision frequency into the parton plasma). We have found that a value of q slightly larger than one characterizes the correlation measure Φ , both by suppressing the boson fluctuations in (15) and by enhancing the importance of the region of large transverse momenta in (14). In the diffusional approximation, a value $q > 1$ implies the presence of a superdiffusion among the constituent particles (the mean square displacement is not linearly proportional to time, having, instead, a power law behavior $\langle x^2 \rangle \propto t^{\alpha}$, with $\alpha > 1$). Effects of anomalous diffusion are strongly related to the presence of non-Markovian memory interactions and colored noise force in the Langevin equation [41]. In this context, we note that in [42], the QCD chiral phase transition is analyzed in analogy with the metal-insulator one and it is shown that an anomalous diffusion regime can occur. The meaning of anomalous diffusion can be found in the presence of multiparticle rescattering, which is very large in Pb + Pb collisions [6, 7].

The early stage of the $p + p$ collision is believed to be radically different from the Pb + Pb case and the influence of the non-extensive statistics appears not negligible in the last reaction. In these terms the anomalous medium effects and the remarkable experimental differences between light and heavy-ion collisions should be better understood.

Finally, we notice that for a given value of the deformation parameter q , the greater the mass is of the detected particle, the more strongly modified are the fluctuations. For this reason, the present calculation predicts more important non-extensive effects for the correlations of heavier mesons and baryons, than for light particles: future measurements of $\Phi_{p_{\perp}}$ for these heavier particles should give clearer evidence of this effect. Furthermore, the non-extensive statistics could imply a sizable modification of the hadronic mass compressibility, since this quantity is strongly related to particle fluctuations (being

more sizably suppressed for particles of heavier masses¹) [43]. We also recall that the non-extensive statistics implies, for $q > 1$, a suppression of the standard boson fluctuations given in (17), which is more effective at high transverse momentum whereas it appears negligible for low momenta. Shuryak suggests [40] that enhanced fluctuations for low p_{\perp} bins, in the pion p_{\perp} histograms, are a possible manifestation of disoriented chiral condensation. We may argue that this effect could be much more obvious if one assumes the validity of the generalized statistics. New data and investigations are necessary to gain a deeper understanding of the high-energy heavy-ion observables.

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¹ Using the same value, $q = 1.02$, of the deformation parameter, we can estimate a reduction of the standard Bose-enhancement factor $1 + \langle n \rangle$ of about 2% for pions, of 7% for kaons and of 11% for ρ mesons.